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Gauge invariant regularization in the AdS/CFT correspondence and ghost D-branes

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Abstract

A field theoretic understanding of how the radial direction in the AdS/CFT correspondence plays the role of a *gauge invariant* measure of energy scale has long been missing. In $SU(N)$ Yang–Mills, a realization of a gauge invariant cutoff has been achieved by embedding the theory in spontaneously broken $SU(N|N)$ gauge theory. With the recent discovery of ghost D-branes an AdS/CFT correspondence version of this scheme is now possible. We show that a very simple construction precisely ties the two pictures together providing a concrete understanding of the radial RG flow on the field theory side.

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The AdS/CFT correspondence [1–3] is the hugely successful conjecture of a duality between large N $\mathcal{N}=4$ super-Yang–Mills theory in four dimensions and type IIB strings/supergravity on anti-de Sitter five-space cross a five-sphere. The field theory is a conformal theory containing an $SU(N)$[1] gauge field, 6 adjoint scalar fields, ϕ^i, and 4 adjoint gauginos, λ. Dilatation symmetries in the theory correspond to rescaling the spatial direction whilst simultaneously rescaling the fields according to their dimension:

$$x^\mu \to \sigma x^\mu, \qquad \phi^i \to \sigma^{-1}\phi^i, \qquad \lambda \to \sigma^{-3/2}\lambda. \tag{1}$$

This symmetry (part of the full $SO(2,4)$ superconformal symmetry) matches in the gravity theory to a symmetry of the spacetime metric

$$ds^2 = \frac{r^2}{R^2}dx_4^2 + \frac{R^2}{r^2}dr^2 + R^2\,d\Omega_5^2, \tag{2}$$

where x_4 are the directions parallel to the D3 world volume and Ω_5 is the metric of a five-sphere. The radius of the space, R, is given by $R^4 = 4\pi g_s N\alpha'^2$ with g_s the string coupling and α' determining the string tension. We observe that under dilatations in the field theory the radial direction, r, transforms as an energy scale. This is a crucial part of the standard correspondence with the radial direction playing the role of renormalization group scale. However, this identification has always been problematic from the point of view of the gauge theory. It is well known that it is very hard to define a gauge invariant energy scale in a field theory essentially because promoting derivatives to covariant derivatives makes their Lorentz invariant length depend on the gauge field.

This problem has been a particularly thorny issue in attempts to generate a Wilsonian description of renormalization group flow in gauge theories. If one cannot define a gauge invariant energy scale how can one follow flow under changes in it? A nice solution has been proposed in [4] and further developed and explored in [5,6]. In these papers the theory is regularized by incorporating the theory into an $SU(N|N)$ gauge theory above the regularizing scale. This theory has been shown to be so restrictive that in the large N limit[1] there are no interactions above the regularization scale—the $SU(N|N)$ theory contains

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[1] In the large N limit we can effectively ignore the difference between $U(N|N)$ and $SU(N|N)$ and any $U(1)$ factors that otherwise need careful treatment [5].

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no dynamics because cancellations between diagrams are exact. The cutoff scale is given by the vev of a scalar field that spontaneously breaks $\mathrm{SU}(N|N)$ to $\mathrm{SU}(N)^2$, thus providing a gauge invariant regularization for the original $\mathrm{SU}(N)$ Yang–Mills. The one-loop and two-loop β function coefficients in Yang–Mills theory have been reproduced using this regulator and moreover without gauge fixing [5,6]. These techniques can be extended to general (perturbative and nonperturbative) calculations in Yang–Mills theory.

It would be nice to make contact between this approach and the gauge invariant energy scale of the AdS/CFT correspondence. We will be able to make this connection here because of the recent discovery of ghost D-branes [7] (also see [8]): combining ghost D-branes with ordinary D-branes allows the construction of $\mathrm{SU}(N|M)$ surface gauge theories. A ghost D-brane is defined by its boundary state being precisely minus that of an ordinary D-brane—they have negative charges and tension. Thus, if N ordinary D-branes are coincident with M ghost D-branes the configuration is the same as if there were $N-M$ ordinary D-branes but with the surface gauge theory being $\mathrm{SU}(N|M)$.

Let us first consider the construction of a 4d $\mathrm{SU}(N|M)$ $\mathcal{N}=4$ gauge theory as the surface theory on N D3 branes and M ghost D3 branes. Each of the $\mathcal{N}=4$ fields is promoted to a supergroup field in the adjoint of $\mathrm{SU}(N|M)$. We may quickly jump to the AdS/CFT correspondence for this theory by noting that the supergravity geometry in the near horizon limit around the stack is just that around $N-M$ D3 branes—it is $AdS_5 \times S^5$ as written above. The gauge invariant operators must match on to supergravity fields as in the usual AdS/CFT correspondence but this matching is a trivial extrapolation because all the same fields exist. The operators are the simple extension of the usual $\mathcal{N}=4$ operators but with the usual trace taken over group indices replaced by a supertrace taken over supergroup indices.

The case where $M=N$ is particularly interesting since then the geometry is that of no D3 branes! The space is simply flat. This is clearly the dual of the complete cancellation of dynamics seen in the field theory for an unbroken $\mathrm{SU}(N|N)$ gauge theory. In our construction to follow, the appearance of flat space will mark the onset of a completely regularized theory.

Next it is interesting to consider configurations where the branes are separated. As usual in the AdS/CFT correspondence the branes can be separated in the 6 transverse directions indicating that the symmetry breaking is associated with vevs for the six scalar fields, ϕ^i. Normally these vevs break $\mathrm{U}(N) \to \mathrm{U}(1)^N$ as the N D3 branes are separated. Separating the D3s must still play this role. Similarly separating the M ghost D3 branes will break $\mathrm{U}(M) \to \mathrm{U}(1)^M$. Of more interest though is the separation of the N D3s and M ghost D3s. If we separate them as blocks then we are clearly breaking $\mathrm{SU}(N|M) \to \mathrm{SU}(N) \times \mathrm{SU}(M)$. This implies the switching on of single scalar with vev in the supergauge space of the form

$$\phi = \Lambda\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & -\mathbb{1} \end{array}\right) + \alpha\Lambda\mathbb{1}, \tag{3}$$

the dimensionless α being fixed by the dynamics [5].

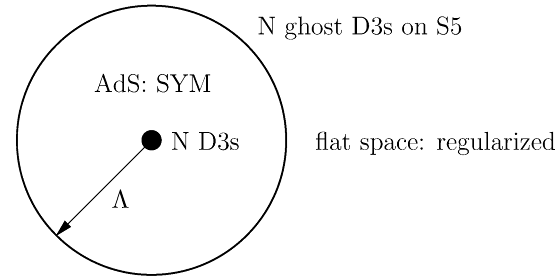

Fig. 1. Sketch of the brane configuration showing regularized $\mathcal{N}=4$ SYM.

The supergravity dual of these set ups will be given by the usual multi-centre solutions

$$ds^2 = H^{-1/2}\, dx_4^2 + H^{1/2}\, dy_6^2 \tag{4}$$

but where H, taking into account the ghosts' negative tensions, is given by

$$H = \sum_{\mathrm{D3}} \frac{4\pi g_s \alpha'}{|y-y_i|^4} - \sum_{\mathrm{ghosts}} \frac{4\pi g_s \alpha'}{|y-y_i|^4}, \tag{5}$$

where y_i are the brane positions.

We now have all the tools necessary to provide an AdS/CFT description of using $\mathrm{SU}(N|N)$ to regularize $\mathrm{SU}(N)$ $\mathcal{N}=4$ Yang–Mills. The construction we will use—see Fig. 1—is a stack of N D3 branes at the origin to generate the field theory we are interested in. We then surround the D3s at a distance Λ by a symmetric shell distribution of N ghost D3 branes on the surface of a five-sphere centred on the D3 branes; at large N we may take the ghost D3 distribution on the five-sphere to be smooth.

The geometry around this set up has two distinct sectors. Within the five-sphere the spacetime is $AdS_5 \times S^5$ because (essentially by Gauss' law) the shell does not contribute. Outside the shell the space is flat since there are the equivalent of no net D3s contained. The cutoff between the two regions is sharp and discontinuous. The AdS space has been cleanly regularized!

Note that stringy corrections will smooth out this transition. Indeed, at energies close to Λ the effects of the long strings stretched between the D3 branes and the ghost branes need to be taken into account. The dynamics of these strings are no longer negligible.

On the field theory side the low energy theory is just $\mathrm{SU}(N)$ $\mathcal{N}=4$ Yang–Mills, the unphysical ghost $\mathrm{SU}(N)$ $\mathcal{N}=4$ Yang–Mills residing in a decoupled sector (the smearing of the ghost branes over the five-sphere breaks the gauge group to $\mathrm{U}(1)^N$), until one moves up to the scale corresponding to a string of length Λ. At this scale the theory becomes the full $\mathrm{SU}(N|N)$ gauge theory and is regularized. In the field theory this transition is naturally smooth. Note, however, that covariant higher derivatives are expected to be required to provide a complete effective cutoff [5]. Such terms are naturally present in the effective field theory description of the stringy corrections.

We can change the regularization scale by moving the spherical shell in the radial direction. It is clear that the radial position of the sphere precisely corresponds to the symmetry breaking scale of the supergroup and hence to a gauge invariant cutoff (a precise measure on the field theory side would be the value of $\mathrm{STr}\,\phi$). This is exactly the identification we sought to make. It is not clear that this regulator is the only one that could be used but it does at least provide a clean field theoretic understanding of the role of the radial distance as a gauge invariant cutoff.

With such a clear gauge invariant regulator in place it should be possible to make more explicit the link between holographic RG flow and the Wilsonian exact renormalization group [9], which clearly must be made via its gauge invariant extension [4,10]. On the field theory side, the AdS/CFT correspondence is the ideal framework to investigate the nonperturbative properties of the proposed $\mathrm{SU}(N|N)$ regularization [4–6]. It is reasonable to hope that with an explicit gauge invariant regulator in place, further progress can be made in understanding nonperturbative aspects of Yang–Mills itself. Importantly, a description of quarks and QCD is possible on both sides of the correspondence [11,12]. Finally, it would be interesting to understand to what extent the string theory and the low energy quantum gravity have themselves been regularized in this framework.

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